2004 IEEE CCA/ISIC/CACSD
September 2, 2004, Taipei

Control Theory:
How Powerful it is for Practical Applications

Masao Ikeda
Osaka University

Contents

1. Introduction

A message of a researcher engaged in control theory for 35 years

- Stability analysis for nonlinear feedback systems with backlash
- Necessary and sufficient conditions for stabilizability of linear time-varying systems
- Overlapping decentralized control of large scale systems
- Two stage design of two-degrees-of-freedom servosystems
- Stability analysis for nonlinear systems with an equilibrium whose location is moved by parameter changes
- Feedforward control of vibration suppression systems
- Dynamic mass measurement of moving vehicles
- Strict LMI approaches for analysis and design of descriptor systems
- Input-output data based design of control systems
- …

A man in control theory thought:

Need to consider philosophy of control theory again

Need to encourage young people in control theory to work in broader fields

Need to be engaged in control applications

2. Control

Realization of desired behaviors in given systems

Desired behaviors:

Regulation: keep a set point ← feedback

Tracking: follow a reference signal

Reference signals:

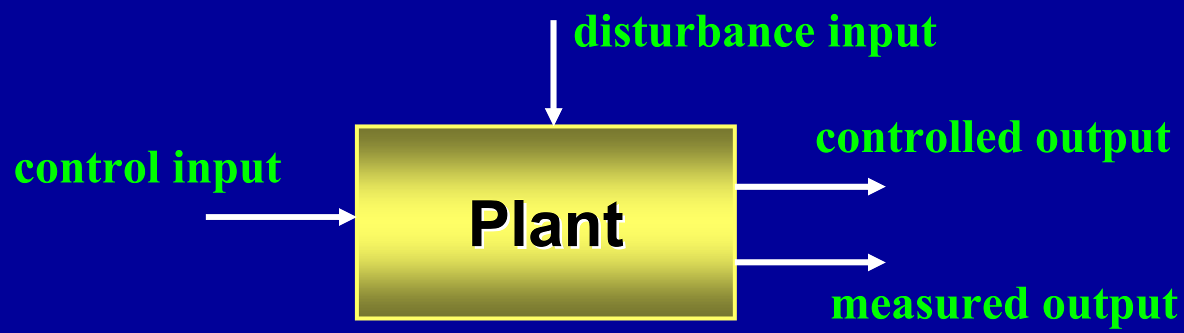
(1) Solutions of LTI differential equations
e.g., step, ramp, sinusoidal, exponential, …

(2) Arbitrary functions, known in advance ← feedforward

(3) Arbitrary functions, unknown in advance ← feedback

We need to apply suitable control laws, i.e., feedback, feedforward, etc.

Plant: System to be controlled



Information for control

Design stage: **plant model (state equations, transfer functions, etc.)**
plant uncertainty, property of disturbance input, …

Operation stage: **measured output, control input, …**

Disturbance input may be measured.
If so, we should utilize the information.

Quality (or difficulty) of control depends on richness of information.

3. Control Theory

Control theory shows (or should show):

(1) what performance we can realize by control

(2) for what kind of plants

(3) under what environmental situation or condition

(4) by what kind of available information, and

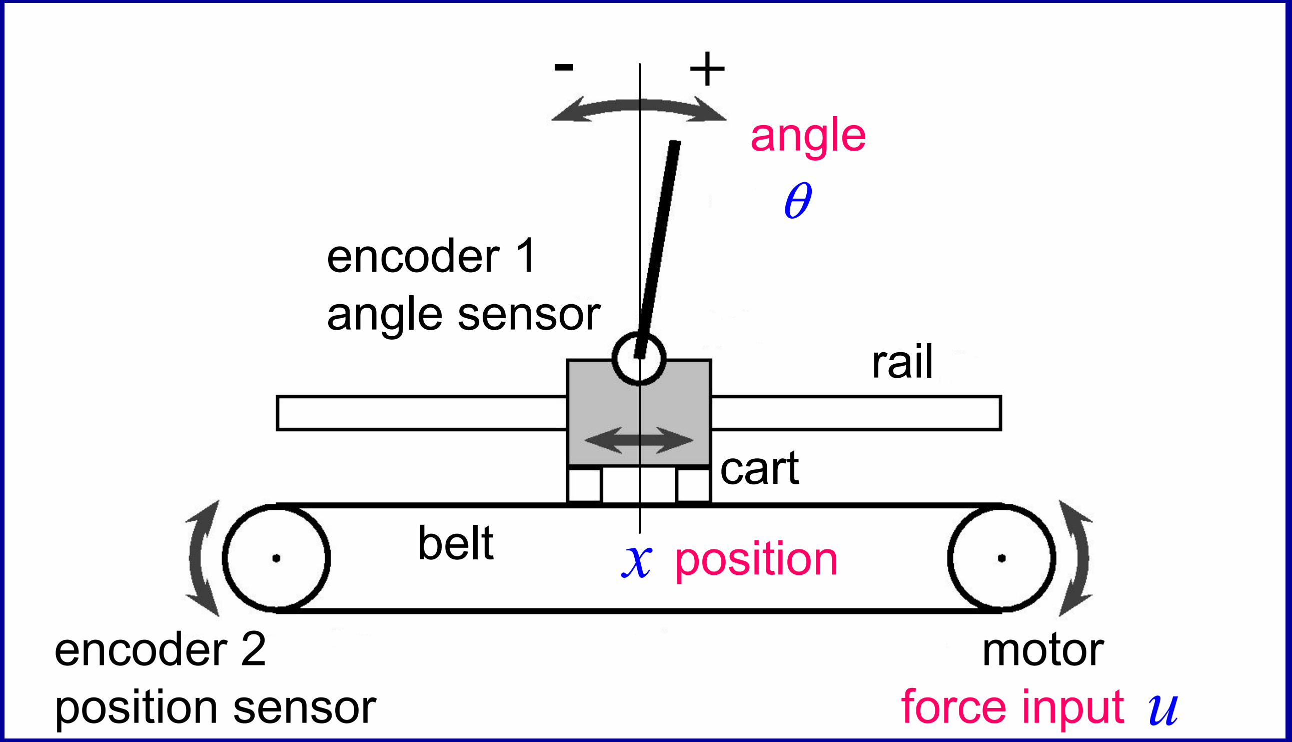
(5) how we should design control laws and operate plants.

Important!!

Control theory also indicates what performance we cannot achieve.

Every control theory should be proposed with the above philosophy.

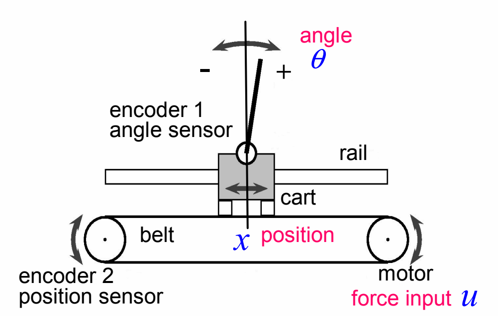
Example: Control of an inverted pendulum



For simplicity of analysis, force input is considered.

Stabilization: $\theta(t) \to 0$ as $t \to \infty$

How do we stabilize the pendulum?



Let us study the behavior of human-being.

Stabilize a stick on the hand.

Then, we imitate the human behavior as follows:

(1) Move the cart to the right fast if θ is positive large and $\dot{\theta}$ is positive.

(2) Move the cart to the right slowly if θ is positive large and $\dot{\theta}$ is negative.

(3) Do not move the cart if θ is positive small and $\dot{\theta}$ is negative.

(4) Do not move the cart if $\theta \approx 0$ and $\dot{\theta} \approx 0$.

(5) Do not move the cart if θ is negative small and $\dot{\theta}$ is positive.

(6) Move the cart to the left slowly if θ is negative large and $\dot{\theta}$ is positive.

(7) Move the cart to the left fast if θ is negative large and $\dot{\theta}$ is negative.

Proposed control law: $u = f(\theta, \dot{\theta})$**:** *static feedback of θ and $\dot{\theta}$*

However, static feedback of θ and $\dot{\theta}$ cannot stabilize the system.

Dynamics of the inverted pendulum system

$$(M+m)\ddot{x}+\nu\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=u$$

$$ml\ddot{x}\cos\theta+(J+ml^2)\ddot{\theta}+\mu\dot{\theta}-mgl\sin\theta=0$$

Control law

$$u=f(\theta,\dot{\theta})$$

M : mass of cart

m : mass of pendulum

J : inertia of pendulum

l : length of pendulum

ν : friction between cart and rail

μ : friction between pendulum and cart

g : acceleration of gravity

Linearized model around $\theta=0$ and $\dot{\theta}=0$ is unstable.

Control theory says:

(1) Static feedback of θ and $\dot{\theta}$ cannot stabilize the system.

(2) Dynamic feedback of θ and $\dot{\theta}$ can stabilize the system.

(3) Static feedback of θ, $\dot{\theta}$, and $\dot{x}$ can stabilize the system.

(4) Static feedback of θ and $\dot{x}+l\dot{\theta}\cos\theta$ can stabilize the system.

$\dot{x}+l\dot{\theta}\cos\theta$: velocity of the top of the pendulum

Question:

Which one is the control strategy of human-being?

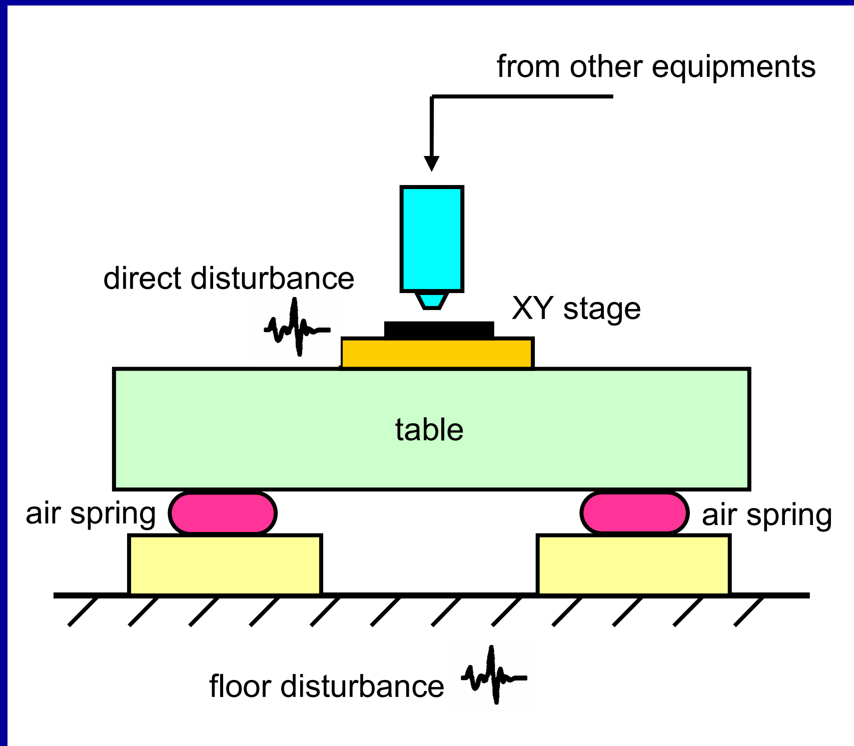
It is not easy to extract the idea of human-being.

4. Vibration Isolation System

Main objective:

Isolation of precision machineries from floor vibration

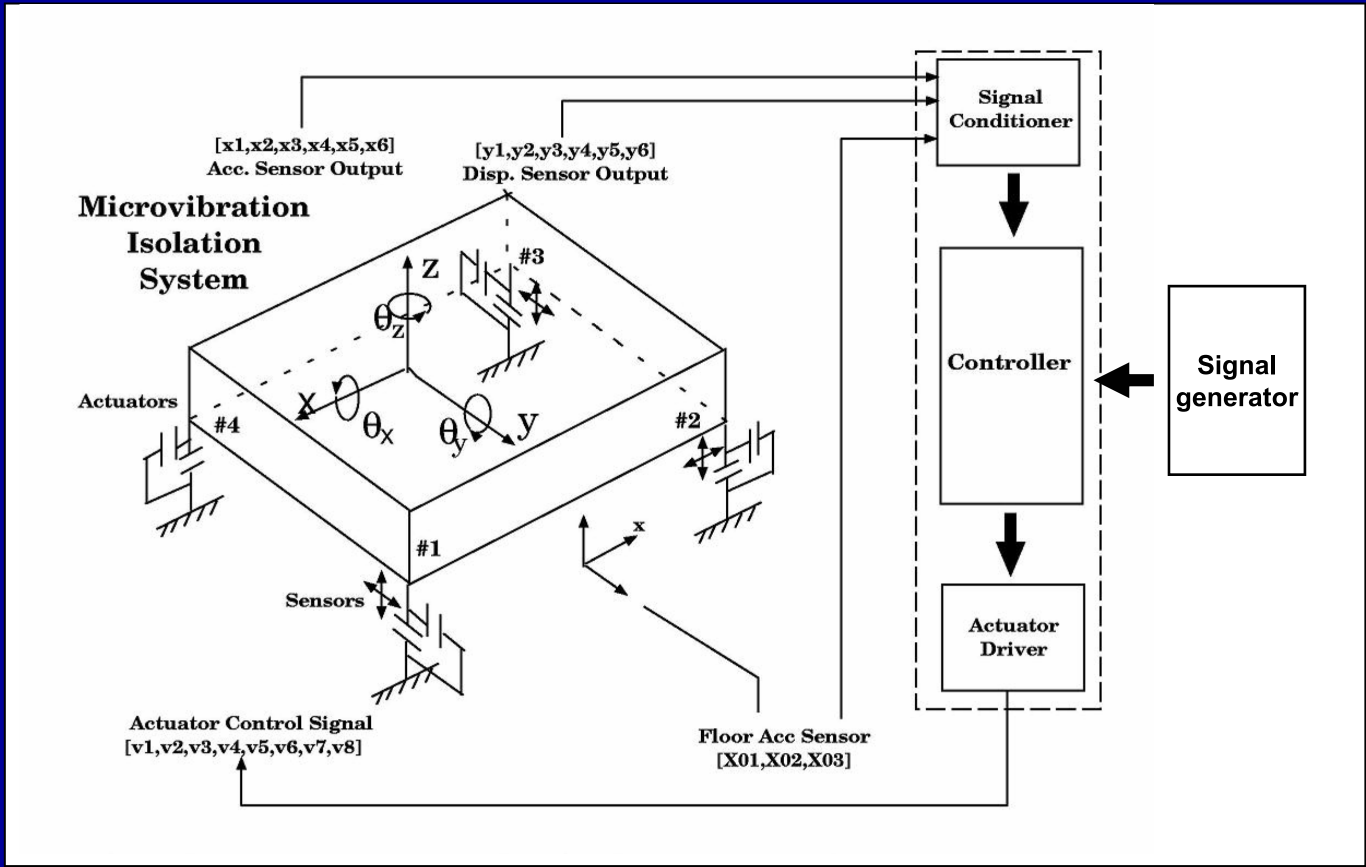
e.g., semiconductor lithography system



Steel table: 220Kg,
700mm x 700mm x 60mm

Supported by air springs

Sensors, actuators, and controller



#1: y, z displacement, acceleration sensors and actuators
#2: x, z actuators
#3: y, z displacement, acceleration sensors and actuators
#4: x, z displacement, acceleration sensors and actuators
floor: x, y, z acceleration sensors

Control objectives and control strategies:

(1) Isolation of the table from floor disturbance

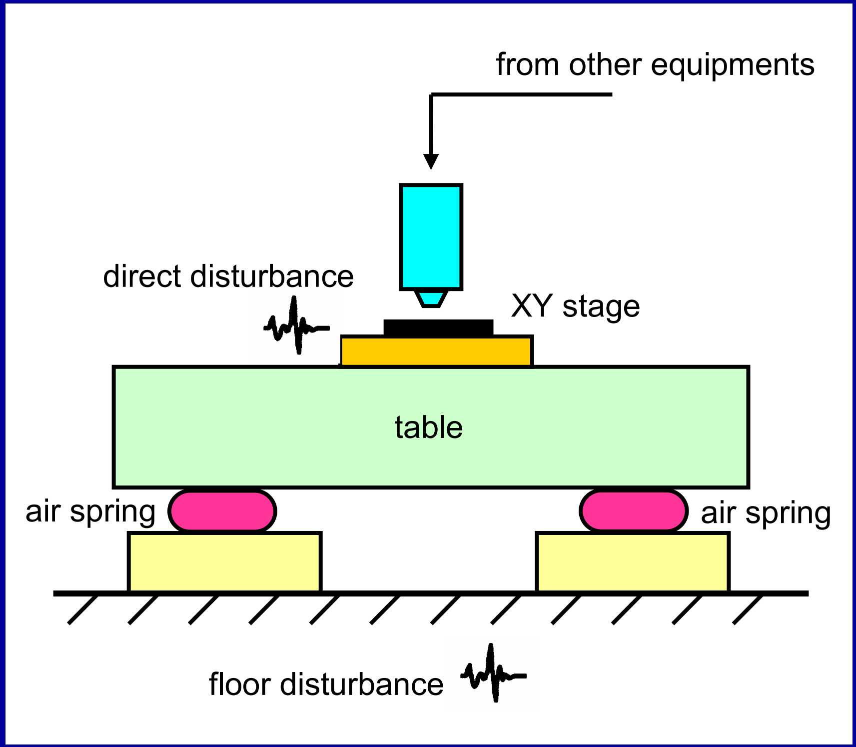
feedback + feedforward of floor disturbance

(2) Suppression of vibration caused by direct disturbance on the table

feedback + control input for cancellation of vibration

(3) Position and attitude control of the table

sensitivity reduction



Note:

For (1), air springs should be soft.
For (2) and (3), air springs should be hard.

Unorthodox control is applied.

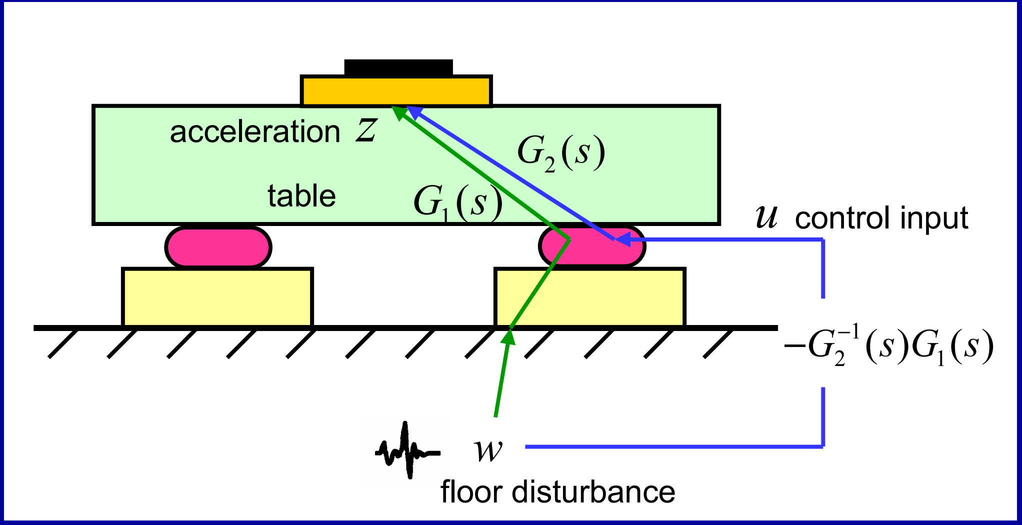
(1) Isolation from floor disturbance

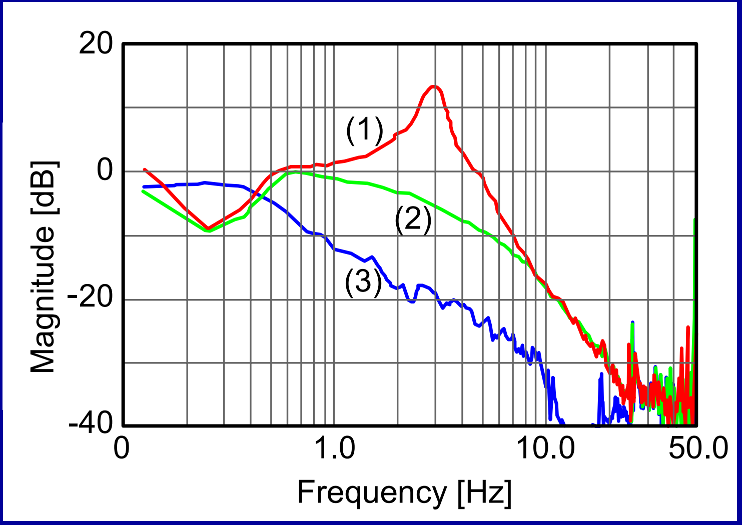
$G_1(s)$: transfer function from floor disturbance w to acceleration z of the table

$G_2(s)$: transfer function from control input u to acceleration z of the table

$$z = G_1(s)w + G_2(s)u$$

Basic idea: cancel $G_1(s)w$ by applying the feedforward input $u = -G_2^{-1}(s)G_1(s)w$





(1) no control
(2) feedback control only
(3) feedback + feedforward control

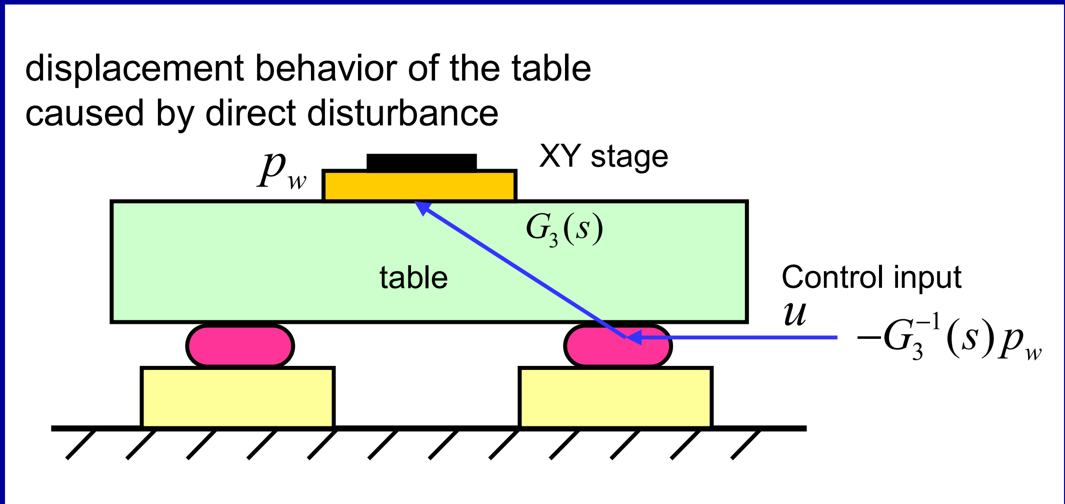
(2) Suppression of vibration caused by direct disturbance

p_w : behavior of the position of the table caused by direct disturbance

$G_3(s)$: transfer function from control input u to the position p of the table

$$p = p_w + G_3(s)u$$

Basic idea: cancel p_w by applying the input signal $u = -G_3^{-1}(s)p_w$



(1) compute the vibration suppression signal in advance

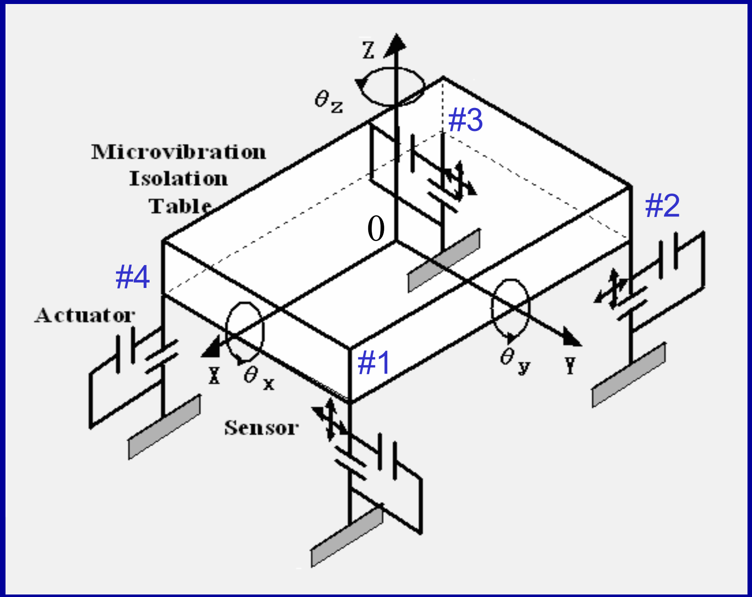
(2) store it in a processor, and

(3) input it when we move the XY stage

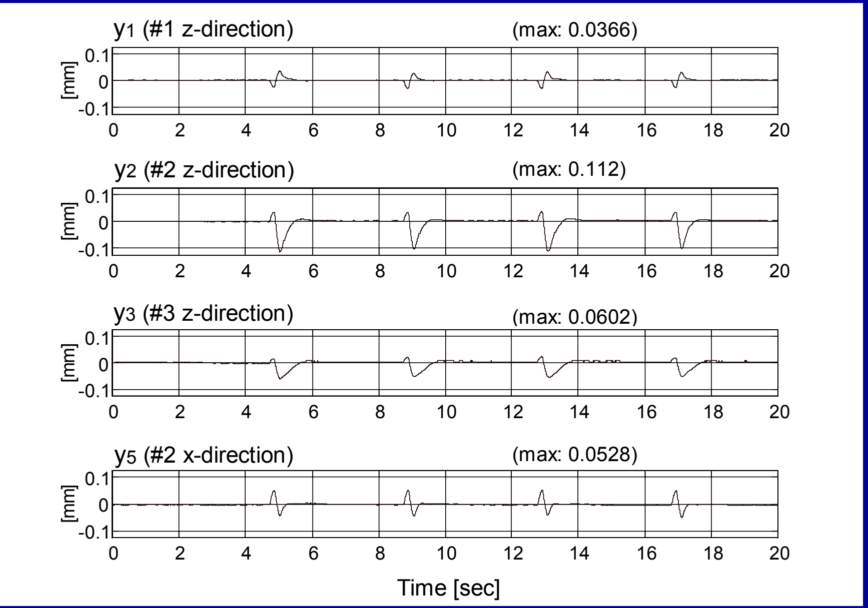
Experimental result

A stage on the table moves to $-x$ direction.

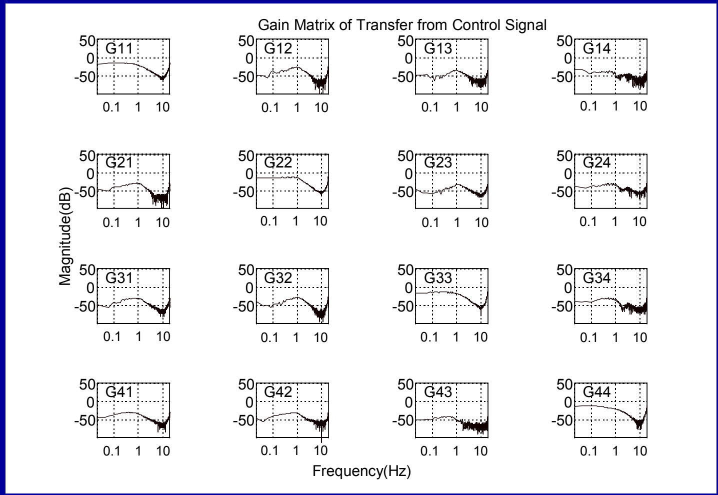
Direction x and rotation θ_y are coupled. Then, the table moves in x and z directions.

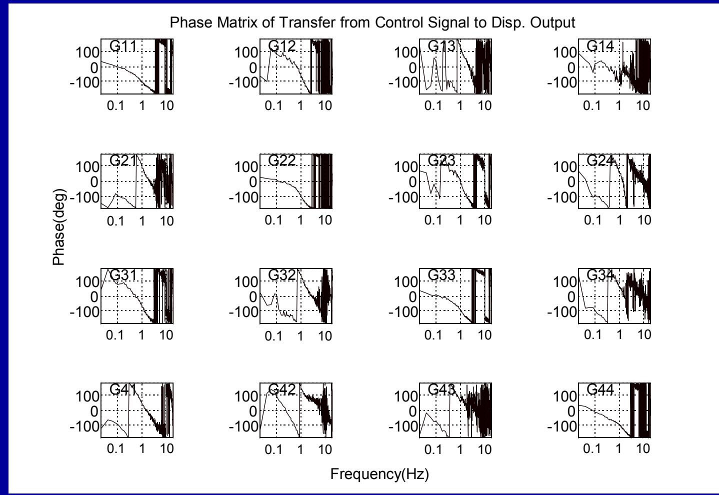


p_w : behavior of the position of the table caused by direct disturbance

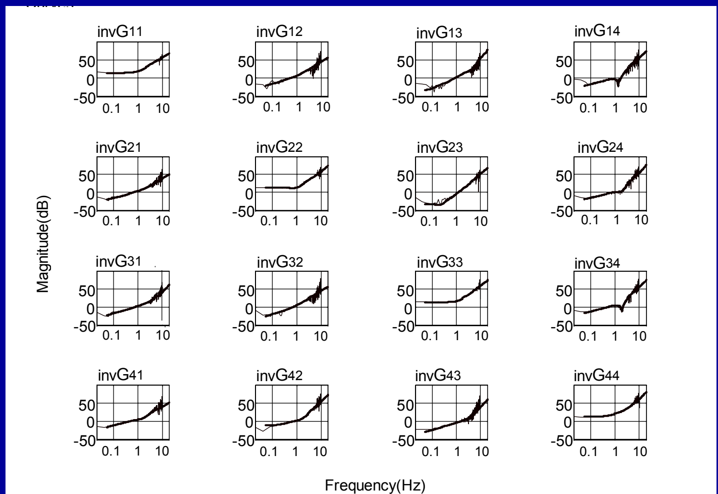


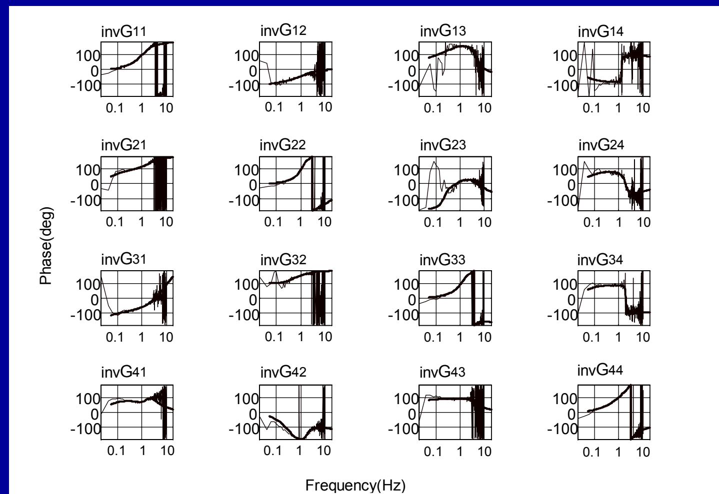
$G_3(j\omega)$: frequency response from control input to the sensor output





$G_3^{-1}(j\omega)$: inverse frequency response





$G_3^{-1}(s)$: inverse transfer function

$$invG_{11}(s) = \frac{s^2+8.99s+32.3}{6.22}$$

$$invG_{12}(s) = \frac{s^2+6.28s+0.363}{-24.8}$$

$$invG_{13}(s) = \frac{s^4-20.5s^3+894s^2+2100s+259}{125s+30000}$$

$$invG_{14}(s) = \frac{s^3+0.153s^2-77.0s-17.8}{-380}$$

$$invG_{21}(s) = \frac{s^2+11.5s+4.36}{49.1}$$

$$invG_{22}(s) = \frac{s^3+63.3s^2+407s+2280}{439}$$

$$invG_{23}(s) = \frac{s^4-43.9s^3-434s^2+566s-836}{766s+30500}$$

$$invG_{24}(s) = \frac{s^4+75.2s^3-301s^2+10300s+3280}{44.4s+42700}$$

$$invG_{31}(s) = \frac{s^4+86.3s^3+3940s^2+41500s+6460}{-199000}$$

$$invG_{32}(s) = \frac{s^2+3.94s-0.136}{24.6}$$

$$invG_{33}(s) = \frac{s^3+40.6s^2+362s+1490}{25.9s+288}$$

$$invG_{34}(s) = \frac{s^3-0.538s^2+142s+28.7}{390}$$

$$invG_{41}(s) = \frac{s^4-1.96s^3-125s^2-2420s-547}{-44.9s^2-838s-5970}$$

$$invG_{42}(s) = \frac{s^4-6.50s^3+93.5s^2-405s+372}{1.38s^2+473s+1460}$$

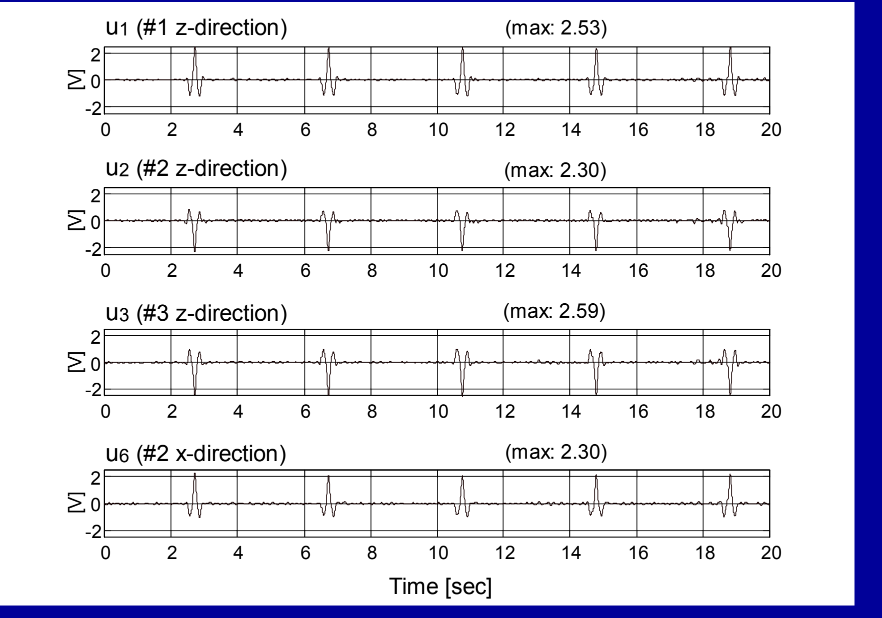
$$invG_{43}(s) = \frac{s^4-30.6s^3+138s^2+23300s+1160}{245000}$$

$$invG_{44}(s) = \frac{s^3+31.3s^2+390s+915}{208}$$

For off-line computation of a finite-time signal $u = -G_3^{-1}(s)p_w$, improperness of the inverse transfer function does not matter.

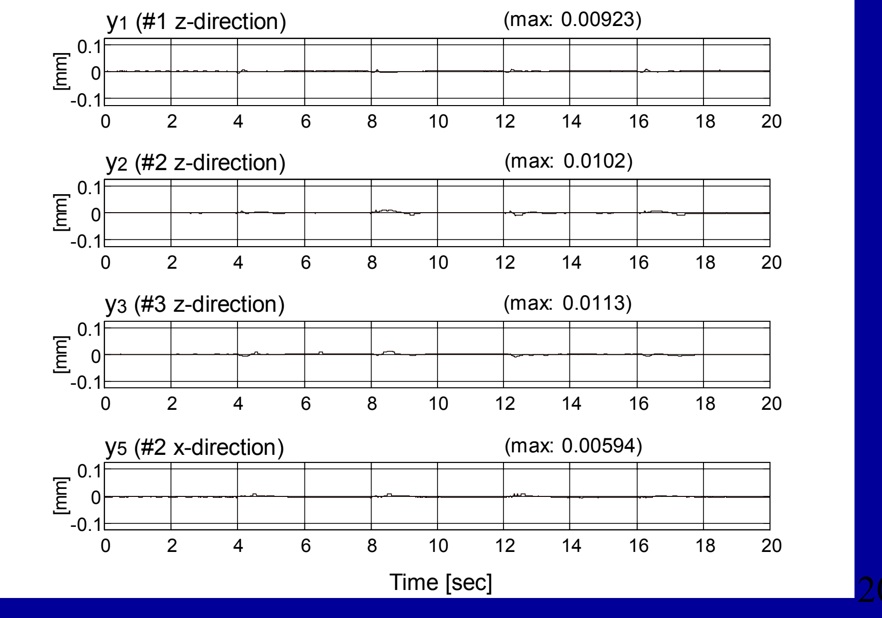
vibration suppression input

$$u = -G_3^{-1}(s)\,p_w$$



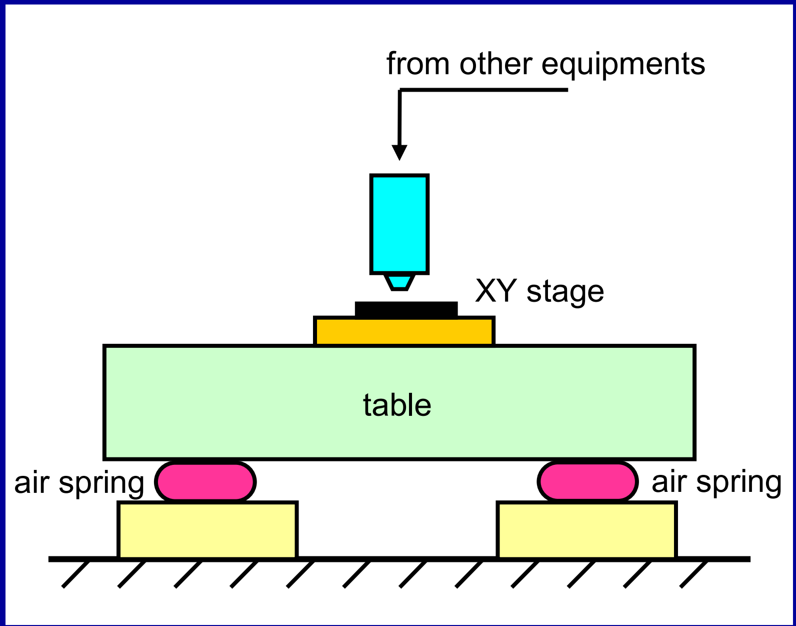
Controlled result:

small vibration
short settling time



(3) Position and attitude control

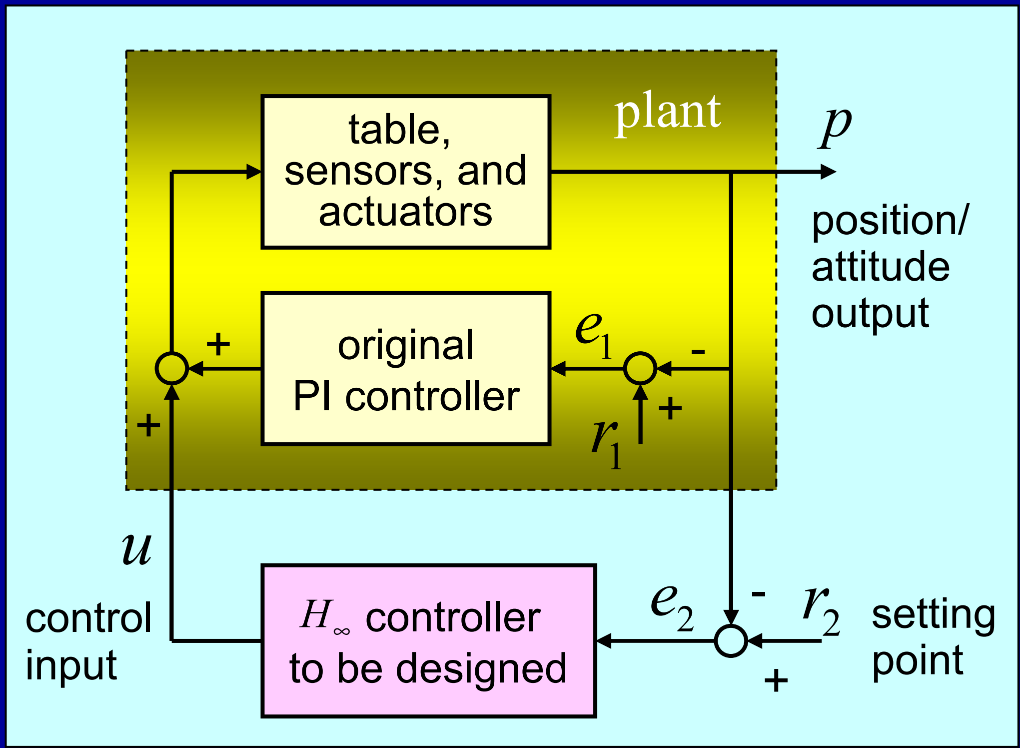
necessary to keep the geographical relation to other equipments



e.g., movement of XY stage changes the position of its center of the mass, and thus the position and attitude of the table supported by air springs.

Normal sensitivity reduction with robustness

Controller design



Original stabilizing PI controller is preserved.

H_∞ controller for sensitivity reduction is designed.

6 degrees of freedom: x, y, z and θ_x, θ_y, θ_z

x and θ_y are strongly coupled.

$$\begin{bmatrix} \frac{-0.3253s^4+25.89s^3+136.3s^2+599.1s+138.8}{s^5+26.67s^4+134.4s^3+422.7s^2+527.9s+176.5} & \frac{-0.01407s^3-11.36s^2+36.23s-20.57}{s^4+7.541s^3+35.57s^2+68.37s+67.32} \\ \frac{4.746s^2-145.4s-50.84}{s^4+47.87s^3+253.4s^2+915.5s+760} & \frac{0.003755s^2-10.95s-3.03}{s^3+6.672s^2+26.99s+17.29} \end{bmatrix}$$

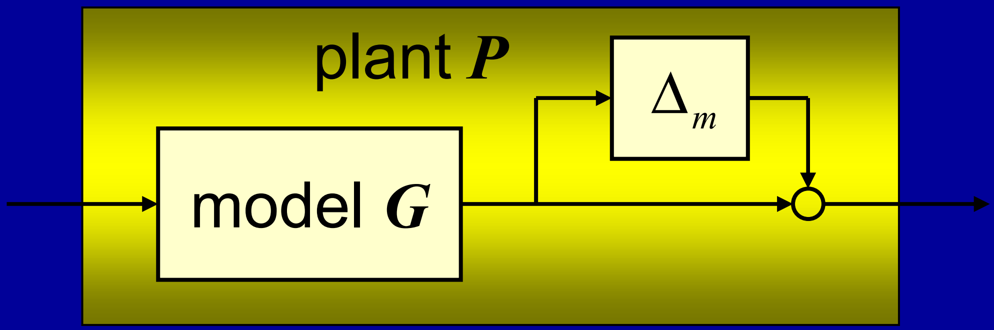
y and θ_x are strongly coupled.

$$\begin{bmatrix} \frac{-0.1064s^3+19.22s^2+182.1s+1193}{s^4+20.92s^3+226.7s^2+642.4s+1029} & \frac{11.52s^2-33.38s+22.11}{s^4+7.663s^3+36.55s^2+75.14s+66.3} \\ \frac{-3.768s^2+157.9s+80.45}{s^4+47.4s^3+263.1s^2+970.4s+1004} & \frac{0.1204s^4-11.36s^3-134.9s^2-850.4s-110.4}{s^5+17.92s^4+185.4s^3+750.7s^2+2320s+916.7} \end{bmatrix}$$

z and θ_z are independent.

$$\frac{-1375s-61.92}{s^4+114.5s^3+667.4s^2+3433s+853.6} \qquad \frac{3555s+1158}{s^4+268.3s^3+3711s^2+3566s+1183}$$

Multiplicative uncertainty

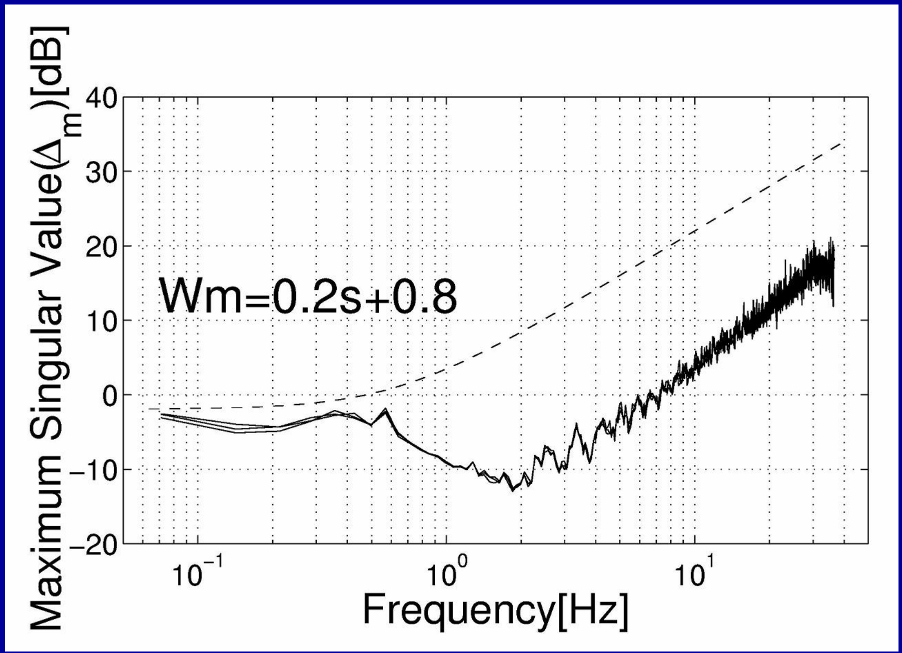


$$\Delta_m = (P - G)G^{-1}$$

P : measured frequency response

G : frequency response of model transfer function

$$P = (I + \Delta_m)G$$

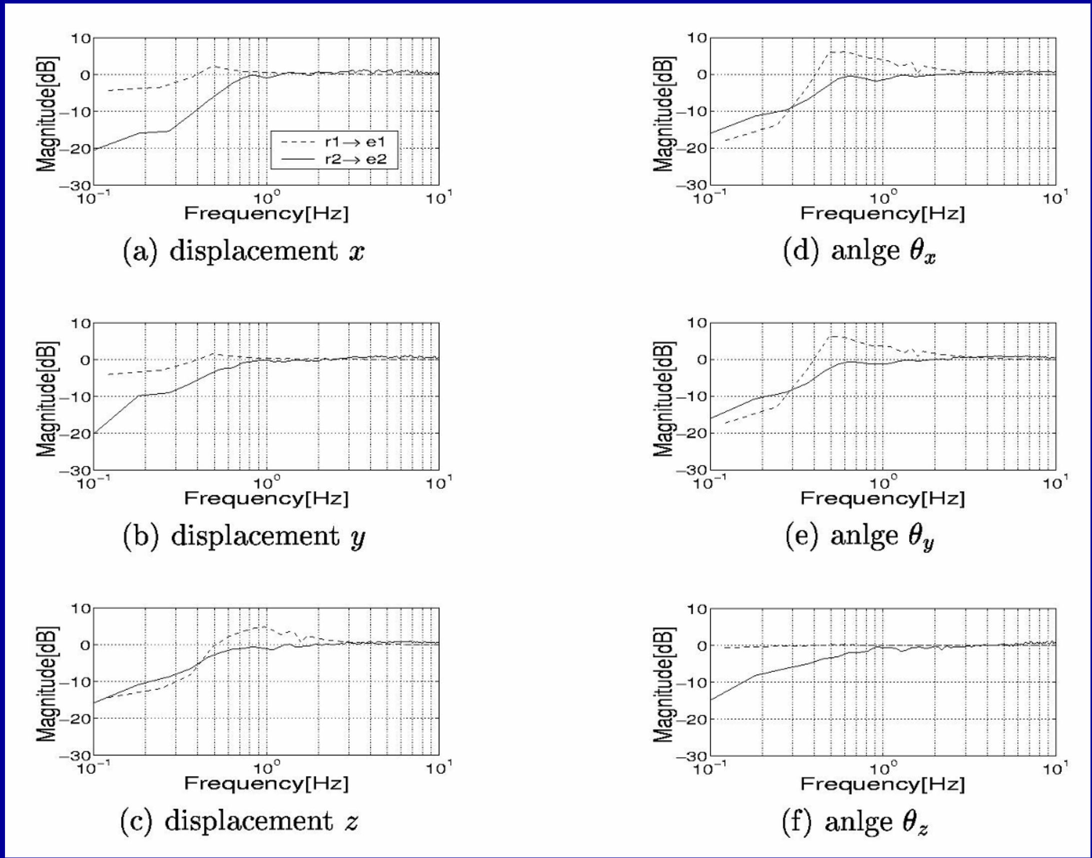


Sensitivity reduction

$$\| \{I + (I + \Delta_m)GK\}^{-1} \|_\infty \rightarrow \min$$

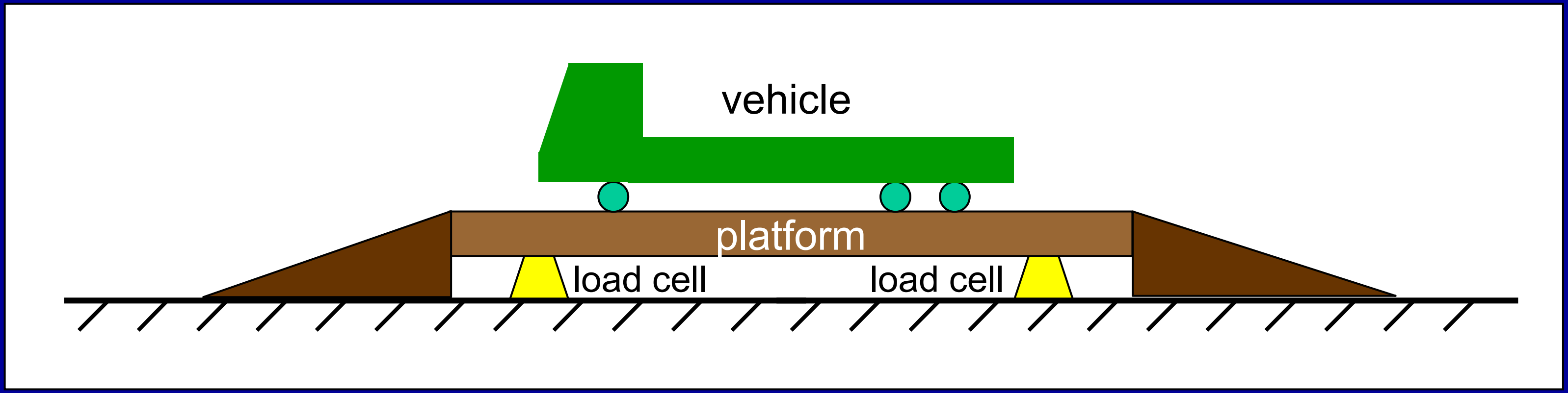
K: controller to be designed

Experimental result



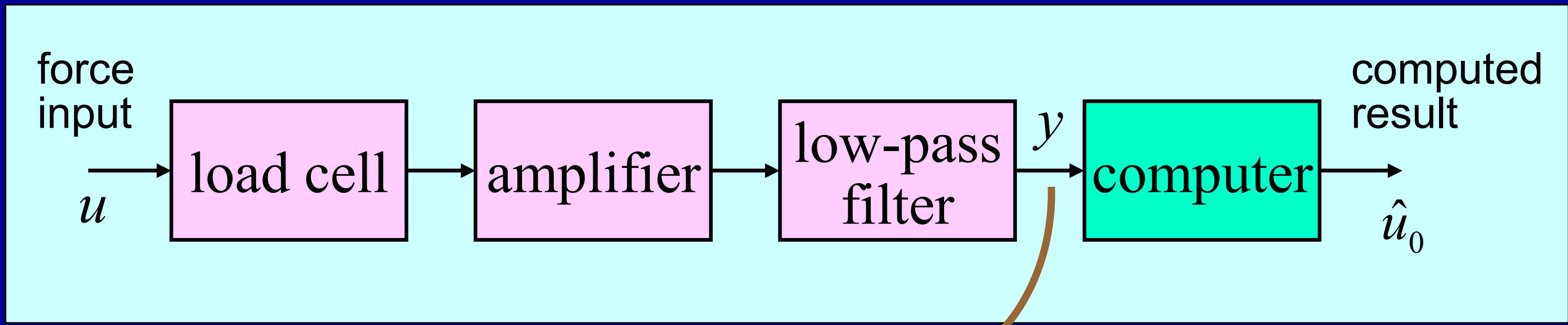
broken line: original PI control only
solid line: proposed control

5. Dynamic Mass Measurement of Moving Vehicles

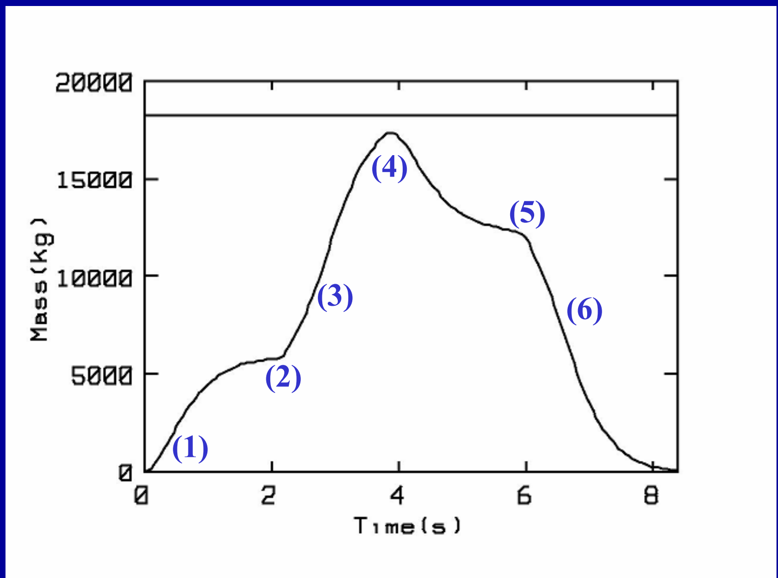


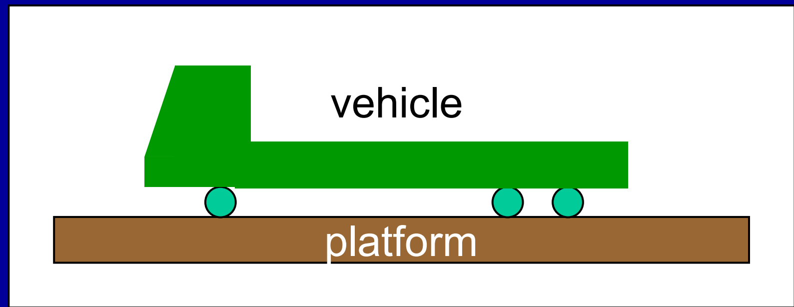
The mass is measured statically. It takes more than 10 seconds to reach the static situation after the vehicle stops.

Measurement system



Low-pass filter output





(1) Front wheels rode on the platform.
(2) First rear wheels rode on the platform.
(3) Second rear wheels rode on the platform.
(4) Front wheels left the platform.
(5) First rear wheels left the platform
(6) Second rear wheels left the platform.

Model of measurement system

from force input u to low-pass filter output y

$$\dot{x}_1(t) = A_1 x_1(t) + b_1 u(t), \qquad y(t) = c_1 x_1(t)$$

Force input : $u(t) = u_0 + u_\delta(t) + v(t)$

u_0 : mass of the vehicle

$u_\delta(t)$: impact force when the vehicle rides on the platform

$= 0, \; t \geq t' > 0$

$v(t)$: force generated by the vertical movement of the vehicle

Vertical movement of the vehicle (unknown)

$$\dot{x}_2(t) = A_2 x_2(t) + B_2 g_\delta(t), \qquad v(t) = c_2 x_2(t)$$

$g_\delta(t)$: disturbance force caused by bumps of road

$= 0, \; t \geq t' > 0$

Total system

$$\dot{x}(t) = Ax(t) + bu_0 + f(t), \qquad y(t) = cx(t)$$

$$A = \begin{bmatrix} A_1 & b_1 c_2 \\ 0 & A_2 \end{bmatrix}, \qquad b = \begin{bmatrix} b_1 \\ 0 \end{bmatrix}, \qquad c = \begin{bmatrix} c_1 & 0 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \qquad f(t) = \begin{cases} \begin{bmatrix} b_1 \\ 0 \end{bmatrix} u_\delta(t) + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} g_\delta(t), & 0 \le t < t' \\ 0, & t \ge t' \end{cases}$$

Measurement data

$$y(t_0),\ \ y(t_0 + T),\ \ y(t_0 + 2T),\ \ y(t_0 + 3T),\ \ldots \qquad t_0 \ge t'$$

$$y(t) = \int_0^t c e^{A(t-\tau)} b u_0 d\tau + \int_0^t c e^{A(t-\tau)} f(t) d\tau$$

$$= c(e^{At} - I)A^{-1} b u_0 + c e^{At} d, \qquad t \ge t'$$

$$d = \int_0^{t'} e^{-A\tau} f(\tau) d\tau \ : \text{constant}$$

$$y_i = y(t_0 + iT)$$

$$\begin{aligned}
y_k + cA^{-1}bu_0 &= cIe^{A(t_0+kT)}(A^{-1}bu_0 + d) && \times a_0 \\
y_{k+1} + cA^{-1}bu_0 &= ce^{AT}e^{A(t_0+kT)}(A^{-1}bu_0 + d) && \times a_1 \\
&\vdots && \vdots \\
y_{k+n-1} + cA^{-1}bu_0 &= ce^{(n-1)AT}e^{A(t_0+kT)}(A^{-1}bu_0 + d) && \times a_{n-1} \\
+ \quad y_{k+n} + cA^{-1}bu_0 &= ce^{nAT}e^{A(t_0+kT)}(A^{-1}bu_0 + d) && \times 1
\end{aligned}$$

$$1y_{k+n} + a_{n-1}y_{k+n-1} + \cdots + a_1 y_{k+1} + a_0 y_k + (1 + a_{n-1} + \cdots + a_1 + a_0)cA^{-1}bu_0 = 0$$

u_0 can be computed from the data $y_k,\ y_{k+1},\ \cdots,\ y_{k+n}$.

Note that

$$1e^{nAT} + a_{n-1}e^{(n-1)AT} + \cdots + a_1 e^{AT} + a_0 I = 0$$

where $\det(\lambda I - e^{AT}) = 1\lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$

Mass can be computed as

$$u_0 = -\frac{y_{k+n} + a_{n-1} y_{k+n-1} + \cdots + a_1 y_{k+1} + a_0 y_k}{(1 + a_{n-1} + \cdots + a_1 + a_0) c A^{-1} b}$$

FIR Filter

Finite Impulse Response Filter

Dynamic mass measurement problem is reduced to:

Parameter estimation of $a_{n-1}, \cdots a_1, a_0,$ and $cA^{-1}b$

$$\det(\lambda I - e^{AT}) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$$

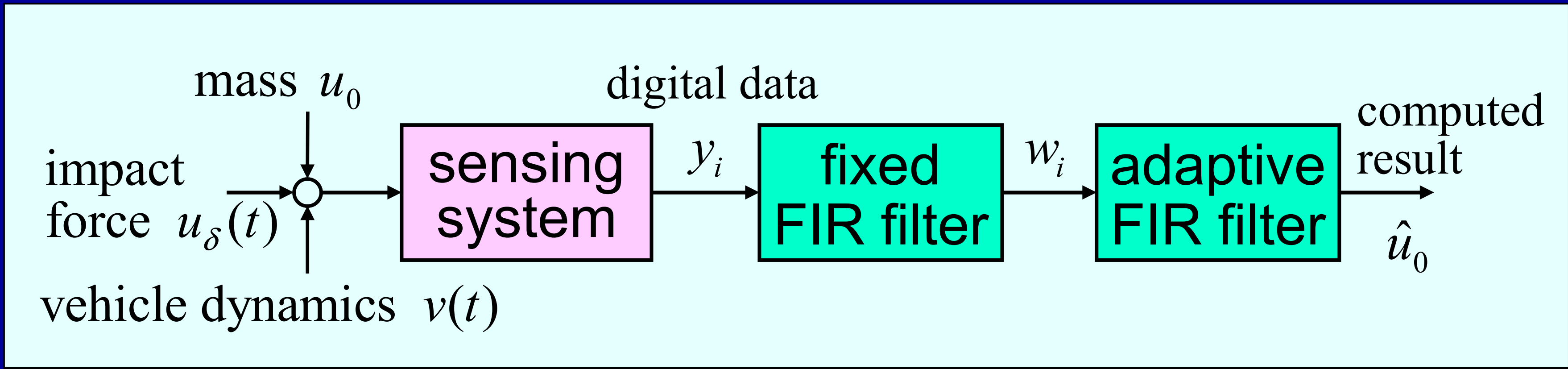
$$= \det(\lambda I - e^{A_1 T}) \det(\lambda I - e^{A_2 T})$$

$\det(\lambda I - e^{A_1 T})$ can be known in advance.

$\det(\lambda I - e^{A_2 T})$ depends on the vehicle to be measured.

$cA^{-1}b = c_1 A_1^{-1} b_1$ can be known in advance.

Dynamic mass measurement system



Fixed FIR filter is designed in advance for the sensing system.

Adaptive FIR filter is tuned by estimating the dynamics of the vehicle from the data w_i. Since the order of the dynamics is unknown in advance, a number of filters with different orders are prepared.

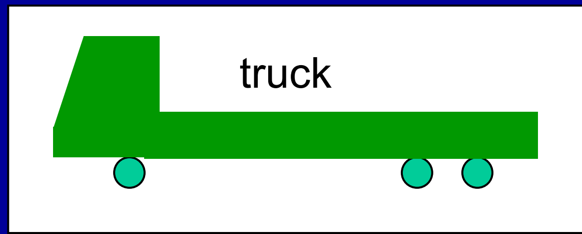
Experimental results

Vehicle: a truck of three axes (one front and two rear)

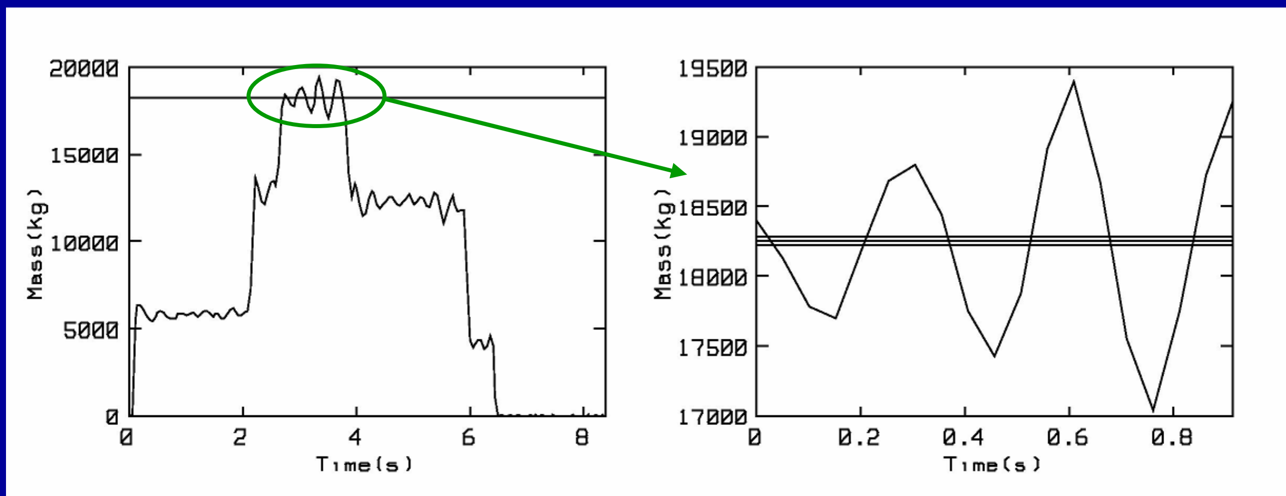
- Distance between front axis and first rear axis: 5.4m
- Distance between two rear axes: 1.3m
- Mass including the driver: 18,255Kg
- Speed of the vehicle: about 12km/h

Platform: 10.5m,

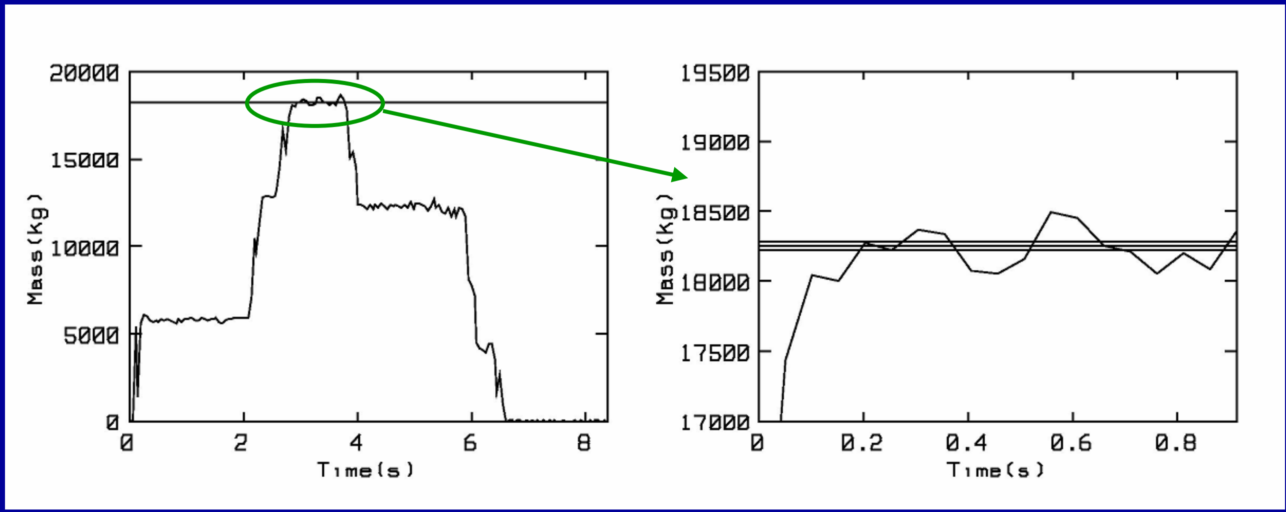
Maximum range of the scale: 30,000Kg



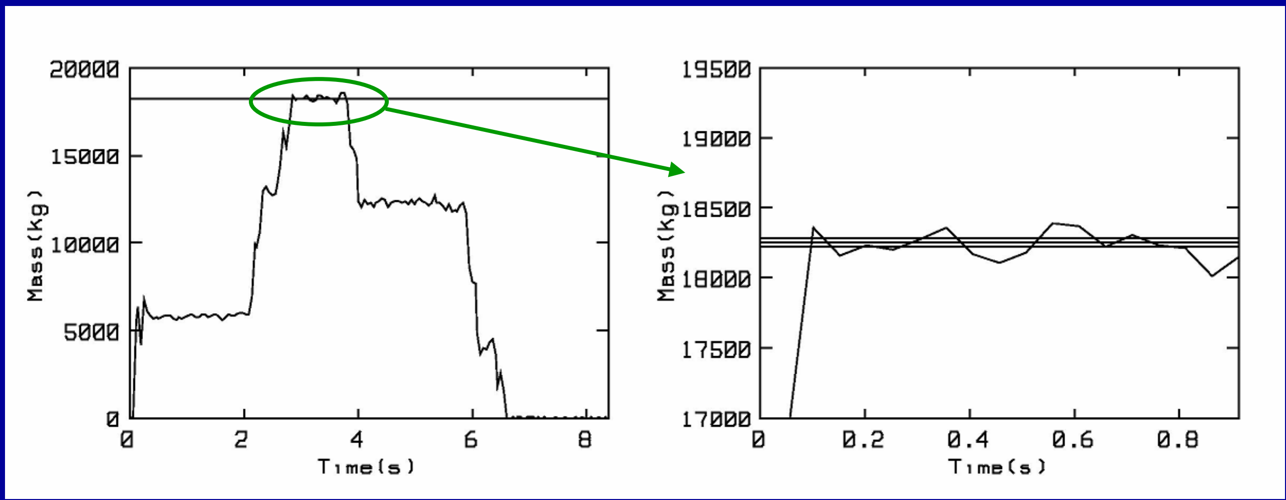
Output w_i of the fixed FIR filter



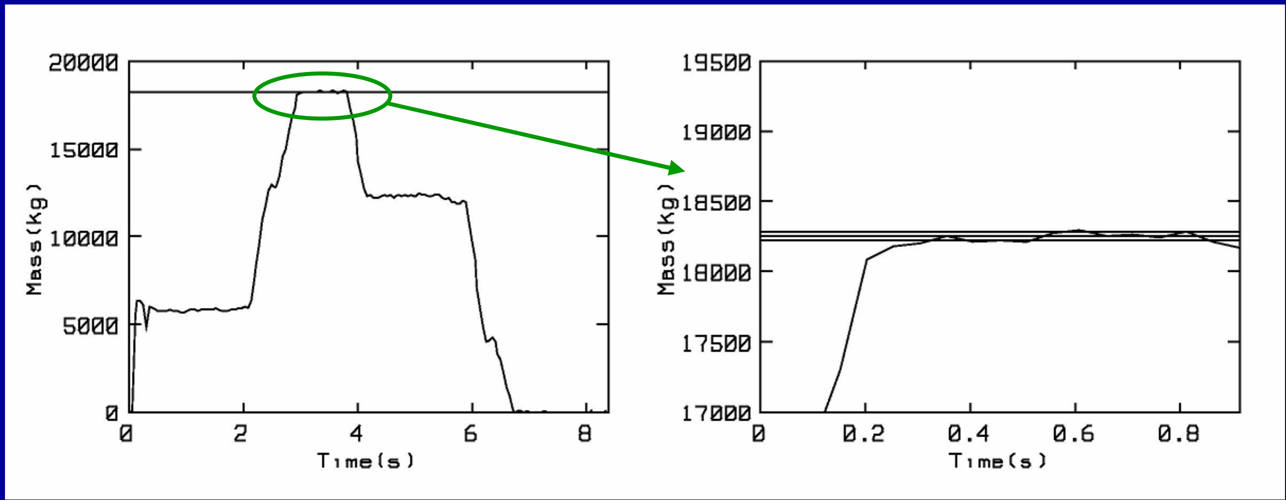
Output $\hat{u}_0$ of the adaptive FIR filter of order 2



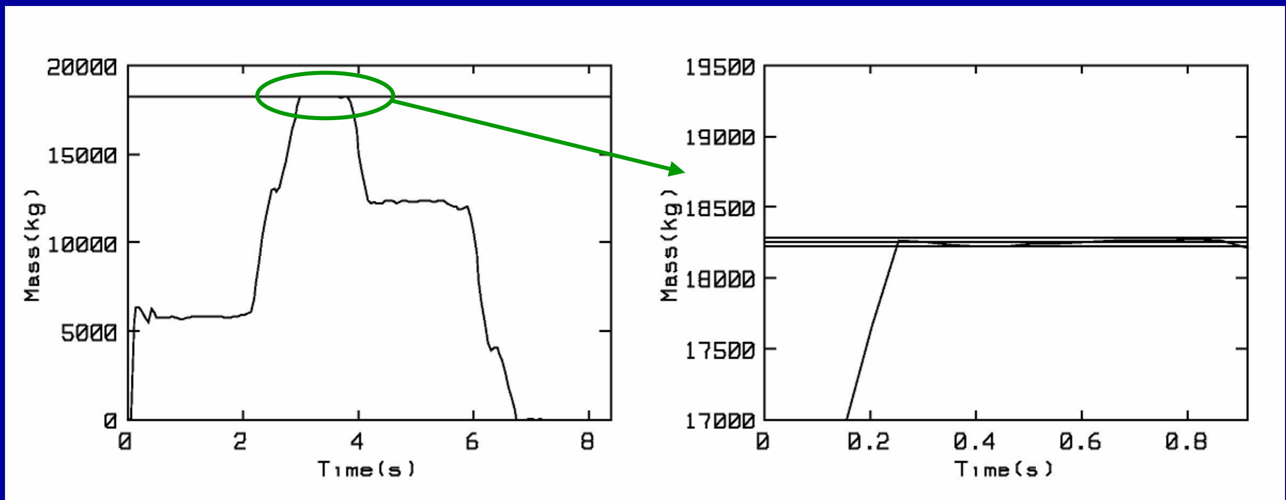
Output $\hat{u}_0$ of the adaptive FIR filter of order 4



Output $\hat{u}_0$ of the adaptive FIR filter of order 6



Output $\hat{u}_0$ of the adaptive FIR filter of order 8



Accuracy 1/1500 was achieved.

In the case when the vehicle speed is about 7km/h, accuracy 1/3000 was achieved.

6. Two-degrees-of-freedom Servosystems

Internal Model Principle:

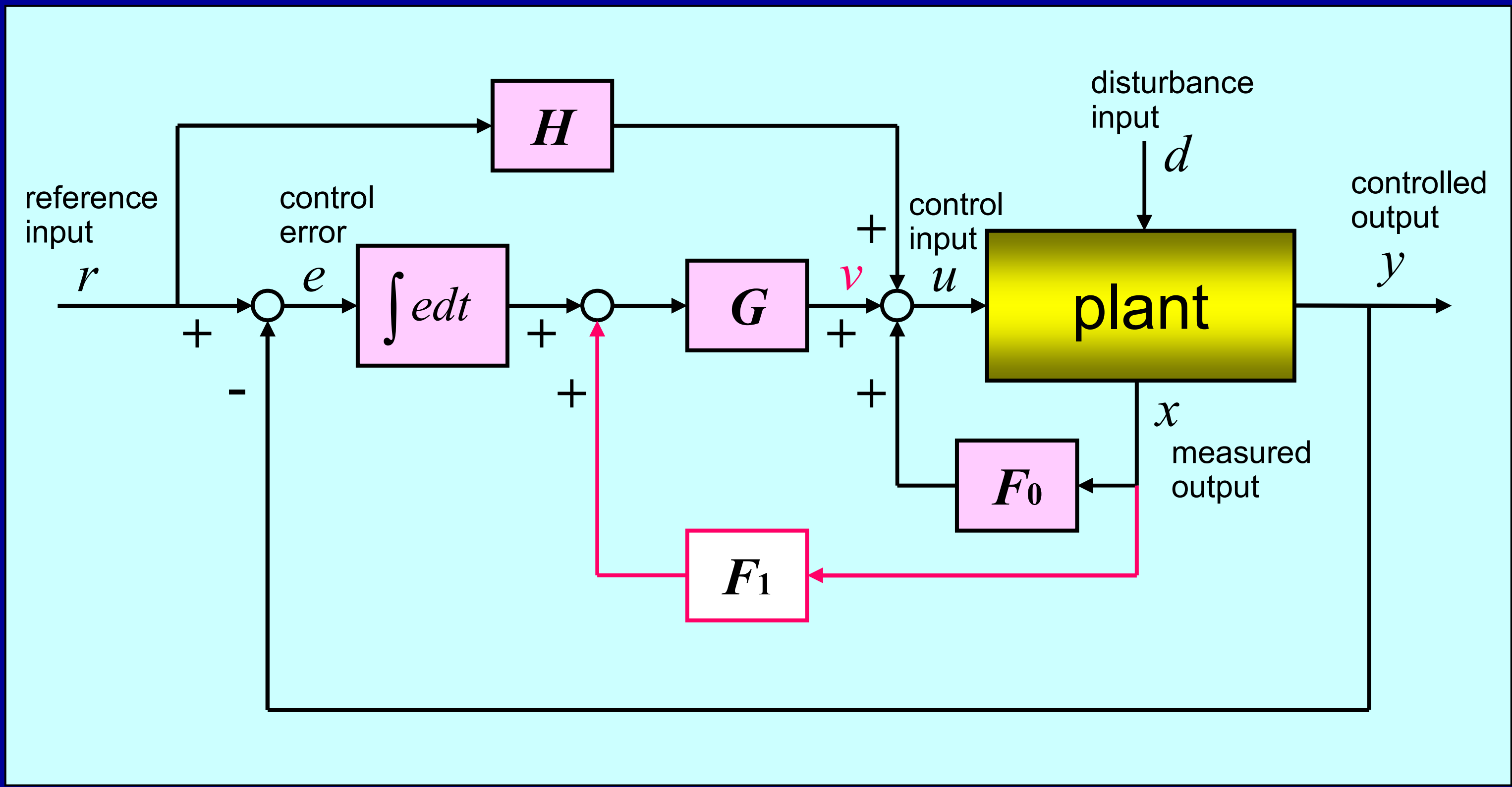
For robust tracking to step reference inputs, an integral compensator is necessary at the control error.

⟹ The integral compensator should be active only when the plant model is not exact or there is a disturbance input.

⟹ A new structure

Applications:

- Control of angle of rear wheels of 4WS cars (HONDA)
- Control of pressure and flow of vapor in boiler systems (Toshiba)
- Semiconductor manufacturing machinery (NEC)
- …



$$\text{plant}: \ \dot{x} = (A + \delta A)x + (B + \delta B)u + Dd, \quad y = (C + \delta C)x + Ed$$

gains : F_0 : stabilizing or optimal gain for the plant

$$H = -\{C(A + BF_0)^{-1} B\}^{-1}$$

$$F_1 = C(A + BF_0)^{-1} \quad \Rightarrow \quad v = 0 \ \text{ when } \ \delta A = 0, \ \delta B = 0, \ \delta C = 0, \ \text{and } d = 0$$

G : stabilizing or optimal gain for the whole system

7. Concluding Remarks

Control theory is very capable, but not enough utilized.

To people in control theory

Control theory should be developed by motivation of practical problems.

People in control theory should look around. There are so many practical problems to which they can make significant contributions.

To people in control applications

Application of a control theory to a practical problem is similar to the relation of a medicine to a disease.

People in control applications need to choose a suitable theory. For this, they should study control theory, not only standard ones prepared by MATLAB.

Control theory needs to learn more from applications.

In 21st Century, control theory has more fields of applications, e.g., in nano technology, bio technology, information technology, ….

People in control theory need to be more flexible.

Acknowledgements:

Advisor:	Prof. Shinzo KODAMA	
Sub-Advisor:	Prof. Hajime MAEDA	
Co-researchers:	Prof. Dragoslav D. SILJAK	Prof. Toshiro ONO
	Dr. Masaki YASUDA	Mr. Takahide OSAKA
	Prof. Jan C. WILLEMS	Prof. Robert E. SKELTON
	Prof. Teruyo WADA	Prof. Kazunori YASUDA
	Prof. Yuzo OHTA	Prof. Takashi KIDA
	Prof. Yukio TADA	Mr. Yoshiro SASANO
Former students:	Prof. Yasumasa FUJISAKI	Dr. David E. WHITE
	Prof. Guisheng ZHAI	Dr. Xiang-Lin TAN
	Prof. Eiho UEZATO	Mr. Nobuo AOKI
	Prof. Eiichi MURAMATSU	Prof. Young-Bok KIM
	Dr. Makoto TAKANO	Dr. Kouji OKUDA
	Mr. Kazuhisa KOUJITANI	Mr. Kiyohiko TSUBOUCHI
	Mr. Naosuke KUNIMOTO	Mr. Takahiro MIURA
	Mr. Naoki KOBAYASHI	Mr. Miyahito HASE